The Moon's Orbit. Practical and computational project

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Abstract

In this workshop we'll tray to calculate the Moon's orbital elements.

For carrying out calculations needed for the whole process, it's necessary to take observations over a few months that enable values of the Moon's orbital elements to be deduced. Sky' observations may be carefully schemed but when a long time is required for our observations weather says 'last word'. If to the weather problem we add to work with students, it's probable that we need a few years for finishing the proposed practice. So, with the end of realising our work in a prudential time, we'll use two different didactic ways.

On the one hand, we would like to work in the traditional way based in observations for calculating orbital parameters with data obtained for this method.

Due to difficulties exposed in the first paragraph, we use a computer' program for obtaining data faster than by observation. All calculations that we have to do for determining parameters can be carried out in the same way.

The computer' program used is the **AstroMath**. It's a *Mathematica 3.0* program that can be used both Macintosh and PC computer.

1 Introduction

Since the Moon is relatively close to the Earth and is held in orbit by the gravitational attraction of the Earth, its orbit is an ellipse with the Earth at one focus. However, because of the ellipsoidal shape of the Earth and the attraction from outside by the Sun and the planets, the Moon's orbit is not a simple ellipse. It is instantaneously an ellipse, but this ellipse is continually changing its shape and orientation around the Earth. This changes are caused by perturbations.

It has required many years of intense investigation by many noted mathematical astronomers to arrive at an equation which permits the computation of the Moon's position at any time. Even now the Moon does not exactly follow predictions, but its place in the orbit must be altered occasionally by the addition of a small term which is determined by observation. Probably the best method of evaluating this term is by occultations of stars.

2 Orbital Elements

As it is well known, a total of six independent parameters are required to describe the motion of a satellite around the Earth. Two of these orbital elements, a and e, describe the form of the orbit, one element T defines the position along the orbit and the three others Ω, i and ω finally define the orientation of the orbit in space. Given these six elements, it is always possible to uniquely calculate the position and velocity vector.

Vice versa there is exactly one set of orbital elements that corresponds to given initial values of r and v, and one may ask how to find these elements. Part of the answer is already evident from the solution of the two body problem.

These elements are defined as follows (Fig.1):

• **a**: is the semi-major axis of the ellipse that the Moon describes in its movement around the Earth.

• **e**: is the eccentricity of that ellipse.

• **T**: is the time elapsed between two consecutive passing of the Moon by the same point, for instance, the perigee (periastrum). (Sideral time: The length of time it takes the Moon to go once around the Earth with respect to the stars. 27,321661 days).

• **i**: the inclination gives the angle intersection between the orbital plane and of the ecliptic. An inclination of more than 90° means that the satllete's motion is retrograde, its direccion of revolution around the Earth being opposite to that of the Earth's rotation.

- Ω: the right ascension of the ascending node indicates the angle between the vernal equinox and the point on the orbit at wich the satellite crosses the ecliptic from south to north.

- ω: the argument of perigee is the angle between the direction of the ascending node and the direction of the perigee.

3 Calculing the orbital elements

We want to calculate the orbital elements with our students. That means we have to use a didactical way for which students could work. Here we propose two complementary ways: the practical-observational and the computational.

Required materials

A star map with degrees marked on the ecliptic; a cross-staff; a photographic camera; a small telescope

3.1 Calculing the **e**

For calculing the eccentricity by a practical way (if we don't have a cross-staff yet: Fig.2) we need a graduated rod (AB in Fig.3). Two metre sticks nailed end to end are suitable. A small piece of wood that can be slid along the rod is also required. A pencil stub attached to the metre sticks with an elastic band is good enough (Fig.3).

The rod is pointed in the direction of the Moon. By trial and error, a position is found for the piece of wood where its angular size equals that of the Moon. Let this position be distance d from the eye.

Then if α and R are the semi-diameter and radius of the Moon (3.476 Km.) respectively, while *2r* is the length of the piece of wood as shown in Fig.3

$$\alpha = r/d = R/D \tag{1}$$

where D is the Moon's distance from the observer.

Let D_P, D_A be the Moon's distances at perigee and apogee. Then

$$D_P = a(1-e) \qquad D_A = a(1+e) \ ,$$

so that

$$\frac{D_P}{D_A} = \frac{1-e}{1+e} \qquad \Rightarrow \qquad e = \frac{D_A - D_P}{D_A + D_P}.$$

But by equation **??** d is proportional to D, so that we may rewrite the e value as follows

$$e = \frac{d_A - d_P}{d_A + d_P}. \tag{2}$$

A large number of readings over many months are taken. A graph of d against time is desired in order to obtain the maximun (d_A) and the minimum (d_P) values. In practice, however, it is found that the intrinsic errors in this crude method make it difficult to draw a suitable curve among the points as plotted in (Fig.4a). Fortunately we know that the variation in distance is periodic and we can use this to enable us to abstract the required information.

From the graph (Fig.4b), values of d_A and d_P can be found, enabling a value of the eccentricity of the Moon's orbit to be calculated from equation **??**.

The method described above requires to take measurements as accurately as possible and a long observational time. It is difficult to obtain a series of observations through a complete lunation, since weather and time of observation often interfere. About that we propose to use photographs for calculating the eccentricity e.

We need two photographs of the Moon. One must be taken when the farther away the Moon is from the Earth; the other, when the distance from our satellite to the Earth is the smallest. Both photographs have to be taken with the same telescope and in the same place. The problem is to know when exactly the Moon is placed in its farthest and closest position from us. We can resolve this problem by two ways: to take photographs during a complete lunation. Since the Moon is a solid body, it is evident that it does not change size. The apparent change must be due to the fact that the distance of the Moon from the Earth is variable. The other way is to use a computer program, for instance the **AstroMath**: by the order **CoordenadasAparentes[Luna, cal[year,month,day]]** we obtain a list with three elements that represent respectively the distance of the Moon to Earth in Km. and the right ascension and the declination in radians. The output can be modified choosing the type of coordinates.

With the photographs showed in Fig.5 and the medium distance (385.000 Km.) to the Moon as datum, we can calculate e.

3.2 Calculing the **a**

With the data obtained by observation for calculating e we can determine a value.

$$D_P + D_A = 2a \qquad \text{and} \qquad D_P = \frac{R}{r} d_P, \quad D_A = \frac{R}{r} d_A \qquad \Rightarrow \qquad a = \frac{D_A + D_P}{2}.$$

3.3 Calculing the **T**

The period is the time the Moon takes to go from perigee to perigee. It is the value of the sideral month.

Observing (Fig.4a) and (Fig.4b), we can find a date for the time of perigee passage from the same graph. It is the date at which the Moon's distance D was a minimum, and therefore the date τ in (Fig.4).

3.4 Calculing the Ω

From the track of the Moon's orbit plotted on the star atlas (Fig.6), it is easy to find a value for the longitud Ω of the ascending node of the orbit on the ecliptic by estimating the angular distance along the ecliptic from γ to **N** (Fig.7). This map can be filled up simoultanetly with the Fig.4.

3.5 Calculing the **i**

To obtain the value of i we need also a star map (Fig.6) where we have plotted different positions of the Moon by observations during at least one lunation. The inclination i is best found by estimating the angular distance between the ecliptic and the orbit, 90° from the nodes (see Fig.7). We can obtain also i by the program computer calculing its ecliptic coordinates at the conveniente date.

3.6 Calculing the ω

The argument of perigee, ω, that is the angular distance between the direction of perigee and the ascending node N, is found by noting on the star chart (Fig.6) the position of perigee. This can be done from our knowledge of the time of perigee passage and from the observations of the Moon's sideral position at known times of observation.

The five elements thus obtained, namely e, i, Ω, ω, τ, give a scale model of the Moon's orbit.

References

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