SECOND, METRE AND SPEED OF LIGHT. HISTORICAL AND EXPERIMENTAL APPROACH

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Abstract

For millenniums, units of time were defined in astronomical terms, using the daily or annual movements of Sun or Earth. Hours were known since the antic Egyptians, minutes and seconds became necessary when the measurements got more and more precise.

Length and distance units were more difficult to define in a universal way. In the eighteenth century, there were two favourite approaches to the definition of the standard unit of length. One of them suggested the use of a pendulum with a half-period of one second. The other defined the metre as one ten-millionth of the distance from the Equator to the North Pole… Astronomic and terrestrial measurements were done for this purpose.

This workshop aims to present some simple experiments, easy to do in the classroom and appointing some of the difficulties about the measurements:

We shall discover some historical documents and experiments related to time and distance measurements, showing the obvious relation between both physical quantities.

We shall do practical experiments about *triangulation* (How could be measured the size of the Earth?), about *pendulum* (Which is the length of the pendulum with a half-period of one second?), and we shall construct an original *correspondence table* between time and (light-) distances using the speed of light as connection.

INTRODUCTION

In astronomy, most of measurements are about space and time. For centuries, scientists had to improve more and more their accuracy, using always more sophisticated technologies.

In fact, one of the oldest measurement units is the *hour*: Egyptians already subdivided daytime and night-time into twelve hours each, since at least 2000 BC, hence their hours varied seasonally. Hipparchus and Ptolemy subdivided the hour sexagesimally and also used a mean hour ($^1/_{24}$ day), but they did not use distinctly named smaller units of time. Instead they used simple fractions of an hour.

The *second* seams to appear around year 1000, when Al Biruni, using again a sexagesimal system, divided each hour into 60 minutes and continued defining *second*, *third* and *fourth* dividing each of them in 60 equal parts. But it was necessary to wait a long time for precise clocks to use currently the *second* as a time unit.

Length measurement used for a very long time such different units as stadiums, king's feet, arms or… thumbs. Only a little more than two centuries ago, during the French Revolution, was defined a new universal unit: the *metre*! There were two favourite approaches to the definition of the new standard unit of length. One suggested to define the metre as one ten-millionth of the length of a quarter of the Earth's meridian. The other proposed to define the *metre* as the length of a pendulum with a half-period of one *second*, thus depending on the time unit.

ABOUT MERIDIAN AND PENDULUM

Even before to come to a decision, many expeditions were sent all around the Earth to do astronomical observations and specially to measure arcs of the terrestrial meridian as well as to determine what length had a "two second period" pendulum.

From 1669 to 1671, **Picard** led a first precise measurement of the meridian in France, using his own triangulation method.

In 1671 **Jean Richer** was sent for an expedition to Cayenne, French Guyana, by the French Government. His first task was to measure the parallax of Mars and deduce from that the distance from Earth to the Sun. But his second important work was to examine the periods of pendulums in Cayenne and to compare his results with those got at other points on the Earth. He found that the same pendulum beat more slowly in Cayenne than in Paris and deduced that gravity was weaker in Cayenne: it was farther from the centre of the Earth than was Paris. Newton and Huyghens used Richer's gravity data to confirm that the Earth was an oblate sphere.

Louis XV, King of France, and the French "Academy of Sciences" ordered two expeditions: one was sent to Lapland, close to the North Pole, under the direction of the Swedish physicist **Anders Celsius** and the French mathematician **Pierre de Maupertuis**: their results with the pendulum proved that gravity grows when going from the Equator towards the Pole.

The other mission was sent to Ecuador, at the Equator. The Ecuatorian expedition left France in May 1735, and arrived to Quito, Ecuador, in June 1736: **Bouguer**, **La Condamine, Godin** and their colleagues measured arcs of the Earth's curvature near the Equator line, from the plains near Quito to the southern city of Cuenca. They completed their survey measurements by 1739, measuring the length of an arc of three degrees near the Equator. However, health problems as well problems related to their astronomical observations kept them in Ecuador several years more…

With pendulum measurements at the top of volcano Pichincha, in Quito and Manta at the sea level, Bouguer found interesting differences. One of them "*0.36 lines between* *the length of the pendulum on the top of Pichincha and at the sea level*" showed that gravity was decreasing dramatically with the altitude, which confirmed Newton's thesis.

Later, in 1751, **Nicolas-Louis de Lacaille** was sent to Cape of Good Hope to prepare a catalogue of austral stars and to measure the Moon parallax. Other important missions were to measure again a meridian arc (he found 57 037 toises, for 1° at latitude 33°18' South) and the length of the famous pendulum!

In 1791, the meridional definition of the *metre* was selected by the French Academy of Sciences. In order to establish a universally accepted foundation for the definition of the *metre*, measurements of this meridian more accurate than those available at that time were imperative. The "Bureau des Longitudes" commissioned an expedition led by **Delambre** and **Méchain**, lasting from 1792 to 1799, which measured the length of the meridian between Dunkerque and Barcelona. This portion of the meridian, which also passes through Paris, was going to be used as a basis for the length of the half meridian, connecting the North Pole to the Equator.

All these measurements enabled the first accurate determination of the size of the Earth and led to the establishment of the international metric system of measurement (SI).

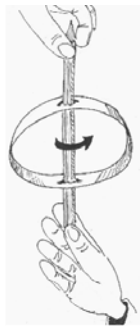
RECREATION

Have a look now at the results obtained by the French expeditions: they measured the length of a portion of meridian with a difference of latitude of one degree. Bouguer and La Condamine, near the Equator (latitude = 0°) got: 56768 "toises". Maupertuis in Lapland (latitude = 70°) got: 57437 "toises".
(One "toise" = 1,94906 metre)

- In your opinion, what is the shape of the Earth?
- Do you confirm Newton's opinion? *Not so easy?*

It is easy to get convinced that the rotation of the Earth causes its flattening. It was not always admitted.

Get a paper band and make a cylinder with it. Place then a pencil as shown on the drawing, and make it spin around its axis. The tape takes a flattened form: the phenomenon can thus be shown easily.



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ABOUT TRIANGULATION

The triangulation method is based on the fact that, knowing the length of one side of a triangle and the values of its angles, it is possible to know the length of the other sides.
This can be done with trigonometric calculations or more easily with a ruler and a protractor, doing well cared constructions.

The difficulty lays in the measurement of the angles. We shall do that in the classroom with quite mediocre accuracy, but using optical material it can be improved a lot.

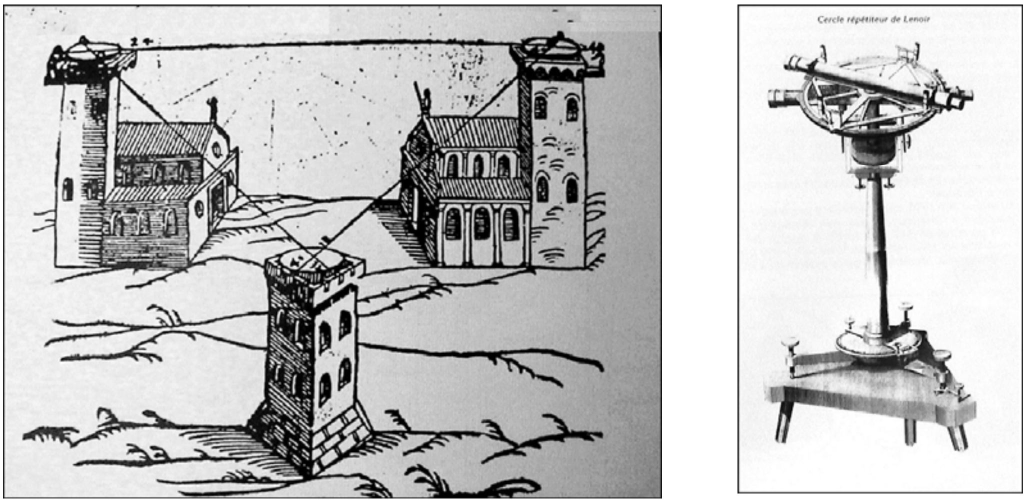


Figure 1

This practical workshop will help you to do “in the classroom” some measurements using the triangulation method, as done by **Delambre** and **Méchain** and so many others (Figure 1) … You can find hereby (Figure 2) a copy of a small part of their measurements along the French meridian.

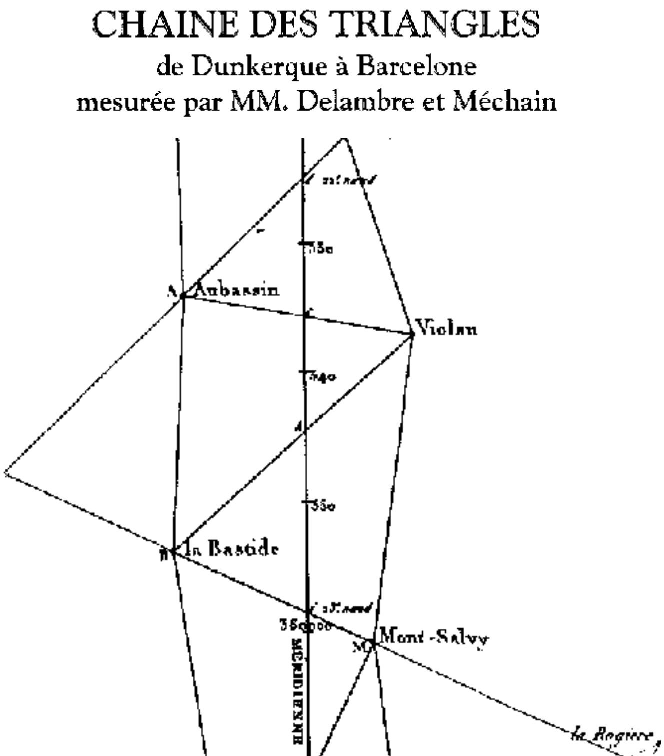


Figure 2

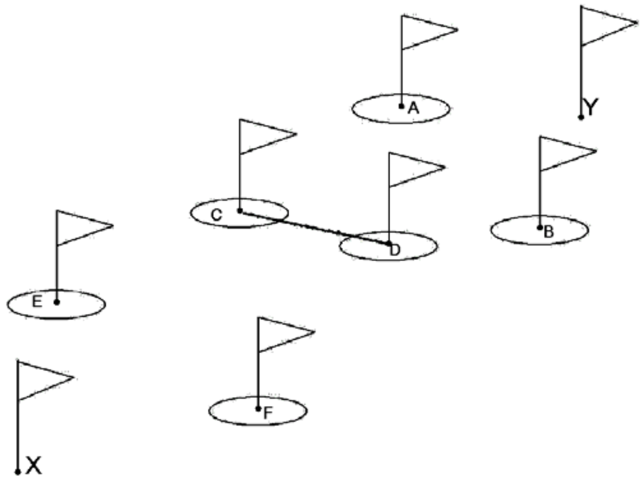


Figure 3

As Delambre and Méchain did with Dunkerque and Barcelona, we shall choose inside the classroom two points X and Y whose distance must be determined. One can use for example one point on the blackboard and another one on the opposite wall (Figure3).

We shall choose also other points in between, such as they did with towers, churches or mountain summits. Let’s give names to each of these points: A, B, C, …

We also need to determine the more accurate value of one of the distances: on the drawing we can assume that distance CD is well known (for example 2,00 metres).

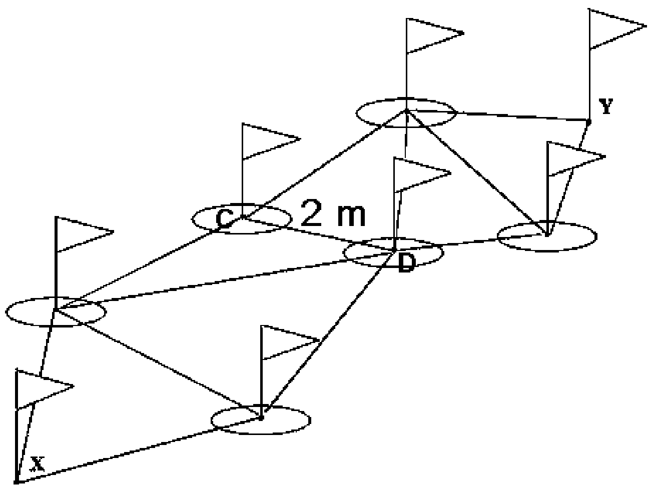


Figure 4

Let’s construct now a virtual network of triangles using all the selected points in the classroom and with X and Y at its ends (Figure 4).

We shall measure all the angles of each triangle, and then it will be possible to construct a scale model of the classroom, with the correct position of all the selected points, and to measure on it the distance between X and Y…

YOUR TOOL

At each point (A, B, C,…) chosen in the classroom, you will place a vertical stick, carrying a little flag with its “name”. This stick will be fixed firmly, just in the centre of a graduated circle (Figure 5). It will then be possible to measure angles from this point, as shown on the photographs.

Good luck!

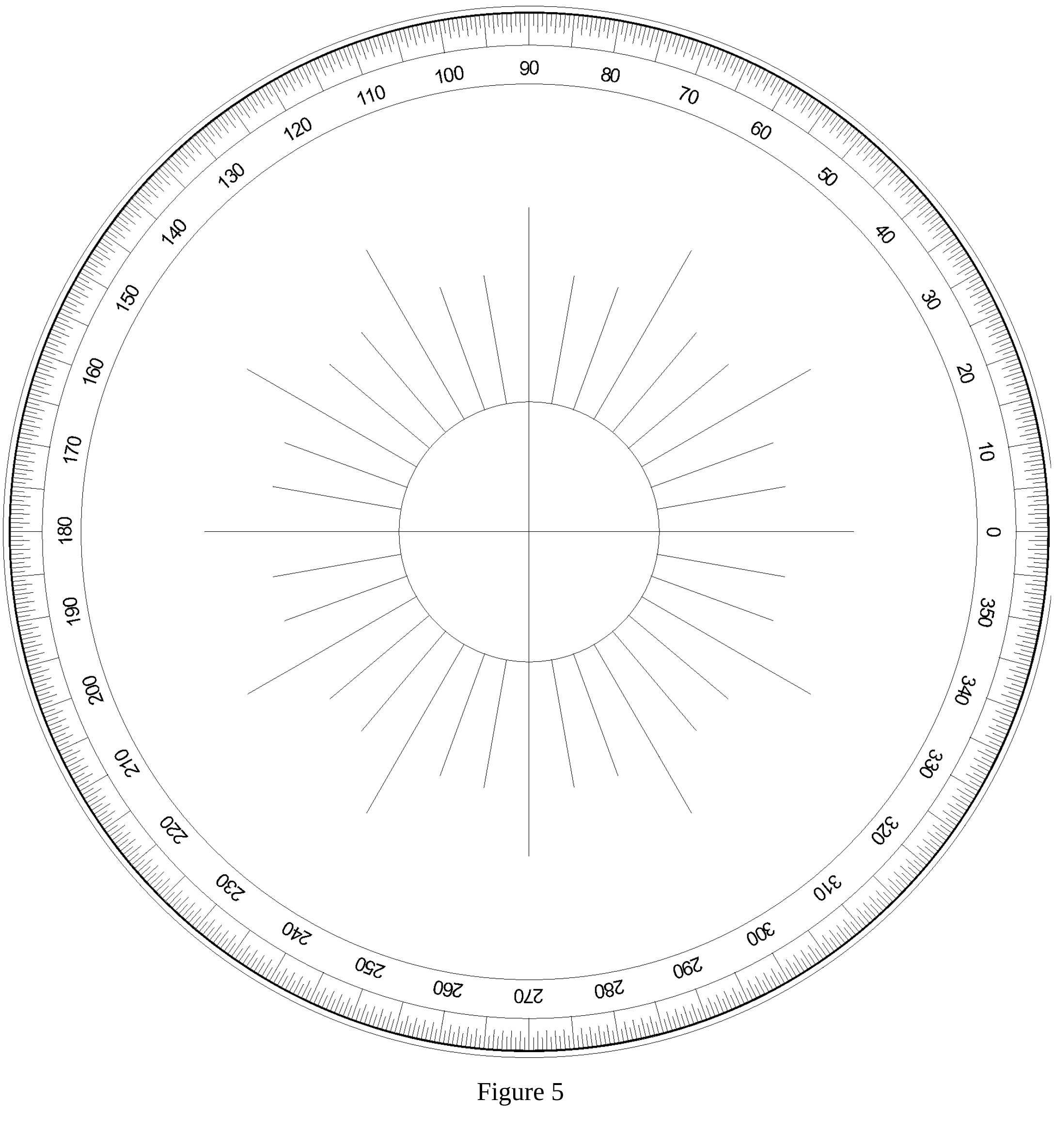
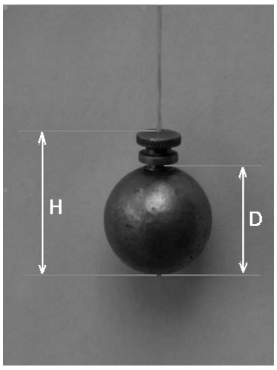


Figure 5

PENDULUM

It is very interesting to investigate with your pupils the problem of the pendulum: *“What is the length of the pendulum with a half-period of one second?”*.

I present here a very simple method conceived by my pupils and their results… I let them speak!



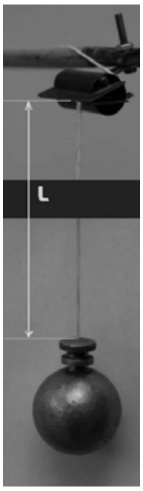


Figure 6

We used 4 simple pendulums whose lengths were close to one metre: two of them shorter than 1 m and the other ones longer. With each pendulum, we measured precisely H, D and L and calculated the length of the pendulum:

$$L + H - D/2 \quad \text{(Figure 6)}$$

With each of our four pendulums, we measured the duration of 100 periods and constructed the graphic giving length against period (Figure 7) and could deduce the answer to our question!

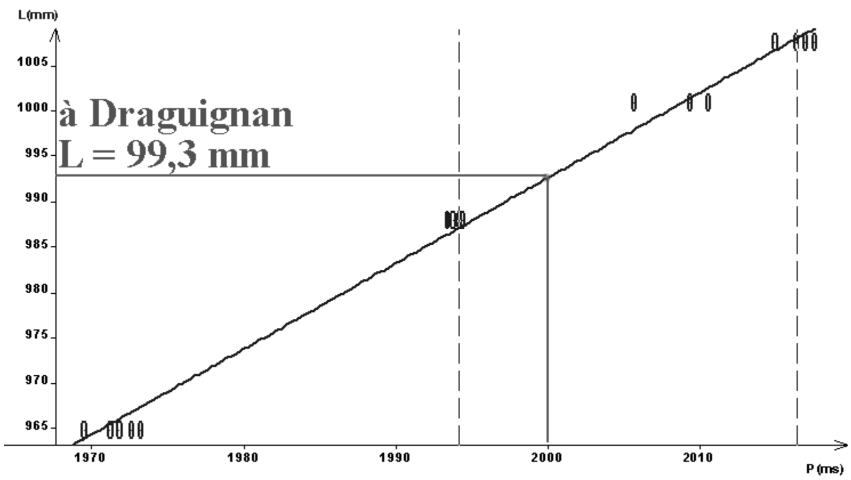


Figure 7

It would be nice to do this experiment in different countries and to compare the results: Will they be different, as numerous scientists said and as seen in the above presentation?

SPEED OF LIGHT

Some information

The first definition of the metre was given during the French revolution: “one ten-millionth of the length of a quarter of the Earth’s circumference”. Since 1983, a new definition assumes that light lasts exactly one second to cover exactly 299 792 458 metres.
Anybody knows that sixty seconds correspond to one minute, and sixty minutes correspond to one hour. One year lasts 365.26 days, each of them lasting 24 exactly hours!
Dinosaurs seem to have disappeared from Earth 65 millions years ago.
Our closest star is obviously … the Sun, at 150 millions kilometres from our Earth, a distance known as Astronomic Unit (A.U.). The age of Sun is estimated to 5 milliards years.
Fortunately, the Moon is close to our planet, only 380 000km, while Pluto orbits at 39 A.U. from the Sun.
α Centauri is the closest star to the Sun, at 4.2 light years (ly) from us, alas invisible from our latitudes.
Vega, in the Lyra constellation lays at 25 ly, while Rigel or Betelgeuse are at 1 400 ly.
Our galaxy, that we call the Galaxy, or the Milky Way, has a 120 000 ly diameter, but the Sun is only at 30 000 ly from its centre.
M31, the only galaxy you can see with the naked eye from Europe, stays at 2 millions ly from us, but that is nothing yet: The Hubble Space Telescope could detect galaxies at several milliard ly from us …

Some calculations

Express in seconds the following durations, with 2 significant numbers.

- One minute:
- One hour:
- One day:
- One year:
- One millennium:
- Time past from the extinction of dinosaurs:
- Age of our Sun:

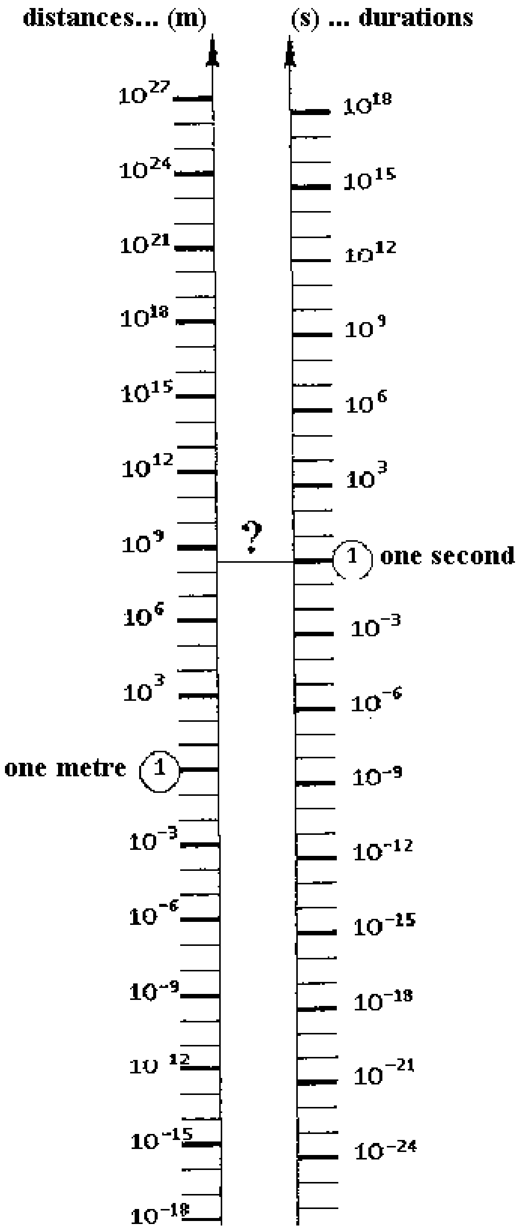
Express in metres the following lengths, with 2 significant numbers.

- Diameter of the Earth:
- Distance Earth-Moon:
- Distance Earth-Sun:
- Diameter of Pluto’s orbit:
- Distance covered by light in one year:
- Distance from the Sun to
    - α Cen:
    - Vega:
    - Rigel:
    - Centre of Galaxy:
    - M31:
    - The end of the known Universe:

And a graphic representation...

Place all these durations and distances on the following diagram:

Space, time and ... speed of light



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