LOCAL ERATOSTHENES EXPERIMENT

How do we calculate Earth's perimeter with our local data?

Eratosthenes measured the circumference of the Earth without leaving Egypt. Eratosthenes' experiment was one of the most and his estimate of the earth's size was accepted for hundreds of years afterwards. It was, in fact, the most accurate estimate until Man was able to go to Space.

Like Eratosthenes, students will make their own measurement of Earth's circumference without leaving their schools.

On June 21st, at noon the Sun will be passing at the zenith of the Tropic of Cancer, which means it will produce no shadow on a vertical stick placed on locations that lie on this parallel. Eratosthenes knew that this was the case of the Ancient Egyptian city of Swenet (known in Greek as Syene, and in the modern day as Aswan) that is located on the tropic of Cancer. This means it has a latitude of 23º 27' N.

Students will use this information as a basis for their calculations.

Determining the distance to the Tropic of Cancer

Eratosthenes is said to have had a slave that measured the distance from Aswan to Alexandria in steps and used an estimated distance between the cities of 5000 stadia (about 800 km if we assume that Eratosthenes used the "Egyptian stadium" of about 157.5 m).

The distance to the Tropic of Cancer can be calculated in several ways. In any of these ways we need a "slave" to measure the distance. It can be a map or an electronic tool.

1) Using a map.

Mark a meridian line from the Tropic of Cancer to your school's location on a map. Then use the scale of the map to make a proportion to obtain the distance.

2) Using Google Earth

On Google Earth:

A. Select **View > Lat/Lon Grid.**

B. Select Tools>Ruler or click to measure length, area, and circumference as follows:.

1. Position the imagery you want to measure within the 3D viewer and make sure you are viewing the earth from top-down (type U) and with terrain turned off for best accuracy. Measuring is calculated using the lat/lon coordinates from point to point and does not consider elevation.
2. From the Tools menu, select Ruler. The Ruler dialog box appears. Consider moving the dialog box to a region of your screen that doesn't obstruct the 3D viewer.
3. Choose the type of shape Line. All versions of Google Earth can measure with Line or Path.
4. Choose the unit of measure for length as kilometers.
5. Click in the 3D viewer to set the beginning point for your line on your school's location and then on the point of the tropic of Cancer that is on the same meridian as your school.
   A red dot indicates the beginning point of your shape, and a yellow line connects to it as you move the mouse.
6. The distance between the two points appears in the box (see Figure 1 bellow).

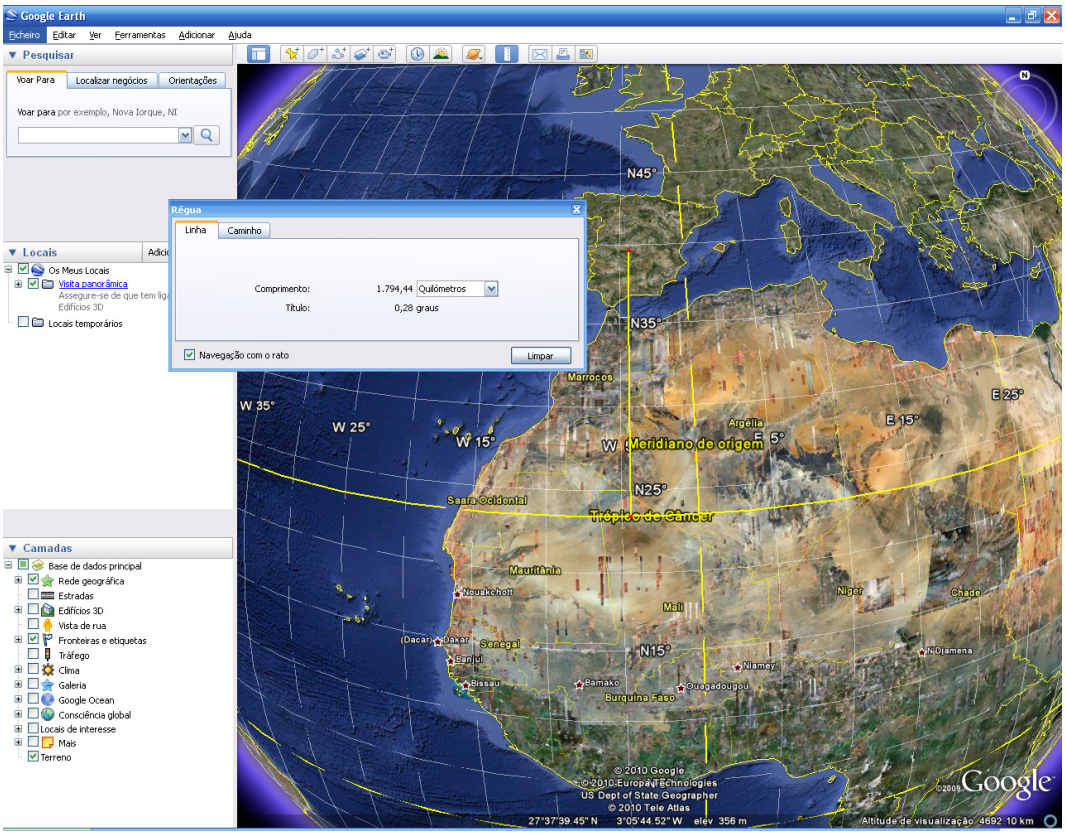

Figure 1.

3) Using the calculator in the project's webpage if you use your exact latitude.

Students can also use the help of the distance to the Tropic of Cancer calculator on the webpage as electronic slave to measure the meridian distance between the Tropic of Cancer and their school.

Students can use GPS for the determination of their latitude and longitude. If you use a GPS you have to convert the latitude in degrees-minutes-seconds to decimal latitude. You can use the latitude converter for this purpose.

If the school doesn't have a GPS receiver then students can use an informatics "slave". Students should use Google Maps (http://maps.google.com/maps) and search for their school's name.

Zoom in until you see your school. Now center the map on you school and press Link on top of the webpage Figure 2.

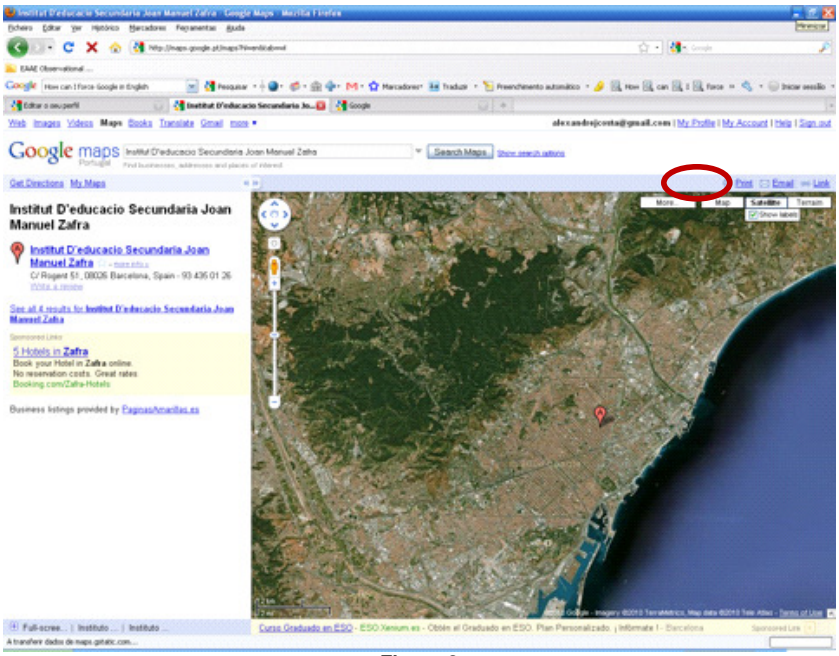

Figure 2

Now copy the HTML associated to this link that appears in the lower box and past it in to a Word or Text document (Figure 3).

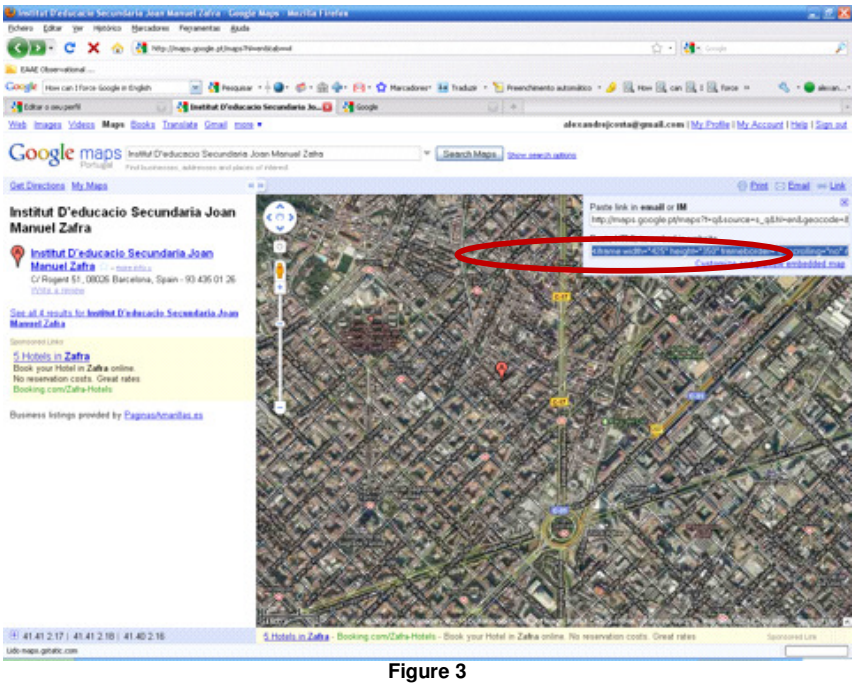

Figure 3

The HTML should contain a text similar to the following:

<iframe width="425" height="350" frameborder="0" scrolling="no" marginheight="0" marginwidth="0" src="http://maps.google.pt/maps?f=q&source=s_q&hl=en&geocode=&q=Institut+D'educacio+Secundaria+Joan+Manuel+Zafra&sll=41.410033,2.15332&sspn=0.175866,0.308647&ie=UTF8&hq=Institut+D'educacio+Secundaria+Joan+Manuel+Zafra&hnear=&ll=41.422907,2.15332&spn=0.175866,0.308647&t=h&output=embed"></iframe>
<small>View Larger Map</small>

In this code ll(red) means latitude, longitude of *map* centre. The digits that are marked in yellow are your approximate latitude and longitude on a decimal basis. Google Maps isn´t accurate enough for the other numbers to have any importance at this point.

Measuring the difference in latitude to the Tropic of Cancer

Now that students have a distance to the Tropic of Cancer the measurement of Earth's Perimeter they need experimental measurements of the shadow of the Sun on the Summer Solstice.

Since the Earth is a sphere than there are some simple trigonometry relationships that allow us to calculate Earth's perimeter.
Sun is so far away from Earth that we can consider that solar rays are parallel (Figure 4).

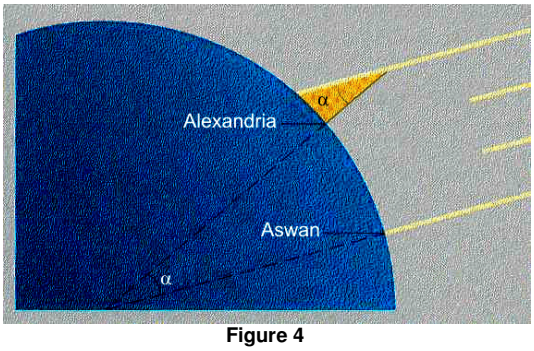

Figure 4

Thus if the Sun is over Siena (Aswan) and procures no shadow at that location it must produce a shadow at Alexandria (or at a school at a different latitude).

Since when two parallel lines are intersected by a transversal line the angles are congruent and we can see that the angle α ate the top of the shadow in Alexandria is equal to the difference in latitude between the two places.

So if we somehow measure the angle at the top of the shadow we can know what is the school's latitude difference compared to the Tropic of Cancer.

The can be directly measured with an angle measuring device but this measurement with the devices available at schools isn't very precise.

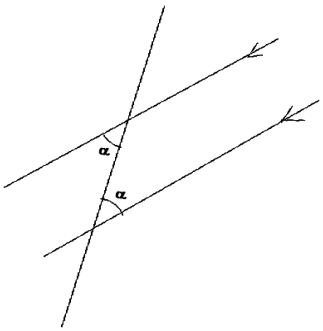

Figure 5. when two parallel lines are intersected by a transversal line the angles α are congruent

It is better to read the height of the stick and the length of the shadow and then determinate the angle (Figure 6). The angle (α) can be obtained by the formula

$$\alpha = \arctan\left(\frac{\text{shadow length}}{\text{stick length}}\right)$$

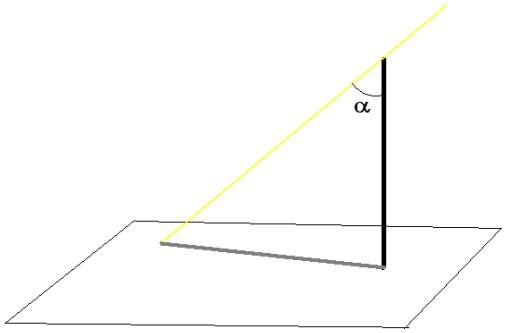

Figure 6

This is the difference in latitude to the Tropic of Cancer and to Aswan. You can use the "Angle at Top calculator" on the webpage to make this calculation.

Measuring Earth's Perimeter

Now all students have to do is reproduce Eratosthenes calculations.

Earth's perimeter can be calculated by the formula

$$P = \frac{d \times 360º}{\alpha}$$

This formula is the outcome of the proportion

$$\frac{d}{P} = \frac{\alpha}{360º}$$

that states the proportionality of distance on the meridian (*d) and the difference in latitude* (α) to the relation between the perimeter (*P*) and the angle of the circle (360º).

We hope you enjoy your determination. Please check your calculations using the "Earth's Perimeter calculator" on the webpage before submitting them to the platform. The accepted Earth's meridional perimeter is 40007,86 km. You are not expected to achieve such a precise measurement. Values between 39000km and 41000 km are acceptable. If you have a value that is not in this interval you probably made a mistake.

Have fun!